

**Assignment for Academic Year 2025-2026 (Session Beginning January 2026)**

**B.A. -2nd Year**

**Course Code: MATH202TH**

**Course Title: Algebra**

**ASSIGNMENT-1**

**Attempt any TWO of the following questions.**

**10 Marks**

**Ques 1.** Let  $Q^+$  be the set of +ve rational numbers. Define  $*$  on  $Q^+$  as under: for  $a, b \in Q^+$ ,  
 $a*b = \frac{ab}{3}$ , verify that  $(Q^+, *)$  is an abelian group.

**Ques 2.** Show that the set  $G = \{0,1,2,3,4,5\}$  is a finite abelian group of order 6 under addition modulo 6.

**Ques 3.** The union of two subgroups of a group is a subgroup iff one is contained in the other. Prove this.

**Ques 4.** If  $H$  and  $K$  are finite subgroups of a group  $G$ , then

$$O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$$

**ASSIGNMENT-2**

**Attempt any TWO of the following questions.**

**10 Marks**

**Ques 1.** State and prove Lagrange's Theorem. Also what is the possible order of a subgroup of a group of order 30? Also list the corresponding no of cosets.

**Ques 2.** A subgroup  $H$  of a group  $G$  is a normal subgroup of  $G$  iff the product of two right cosets of  $H$  in  $G$  is again a right coset of  $H$  in  $G$ .

**Ques 3.** Prove that if  $H$  is a subgroup of an abelian group  $G$ , then the group  $G/H$  of all right cosets of  $H$  in  $G$  forms an abelian group under the composition defined by  $Ha.Hb = Hab$ .

**Ques 4.** Prove that the set  $G = a + \sqrt{2}b$ ,  $a, b \in Q$  where  $Q$  is the set of rational number, is a ring.

**ASSIGNMENT-3**

**Attempt any TWO of the following questions.**

**10 Marks**

**Ques 1.** Show that the set  $Z_n = \{1, 1, 2, 3, 4, 5\}$  is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.

**Ques 2.** Show that the ring  $Z_7$  is a ring without zero divisor. Also find the units of  $Z_7$ , which is commutative ring with unity.

**Ques 3.** Every field is an integral domain. Prove this.

**Ques 4.** Prove that  $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$  is a nilpotent element of  $M_3(\mathbb{R})$ , of real matrices of order  $3 \times 3$ .