Assignment For Academic Year 2024-25 (Beginning January 2025)

B.A. -2nd Year **Course Code: MATH202TH** **Course Title: Algebra**

ASSIGNMENT-1

Attempt any TWO of the following questions. **10 Marks**

Ques 1. Let Q^+ be the set of +ve rational numbers. Define * on Q^+ as under: for a, b $\in$ Q+, $a*b = \frac{ab}{3}$, verify that $(Q^+, *)$ is an abelian group.

Ques 2. Show that the set $G = \{0,1,2,3,4,5\}$ is a finite abelian group of order 6 under addition modulo 6.

Ques 3. The union of two subgroups of a group is a subgroup iff one is contained in the other. Prove this.

Ques 4. If H and K are finite subgroups of a group G, then

$$O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$$

ASSIGNMENT-2

Attempt any TWO of the following questions. **10 Marks**

Ques 1. State and prove Lagrange's Theorem. Also what is the posible order of a subgroup of a group of order 30? Also list the corresponding no of cosets.

Ques 2. A subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

Ques 3. Prove that if H is a subgroup of an abelian group G, then the group G/H of all right cosets of H in G forms on abelian group under the composition defined by Ha.Hb = Hab.

Ques 4. Prove that the set $G = a+\sqrt{2b}$, a, b $\in$ Q where Q is the set of rational number, is a ring.

ASSIGNMENT-3

Attempt any TWO of the following questions. **10 Marks**

Ques 1. Show that the set $Z_n = \{1, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.

Ques 2. Show that the ring Z_7 is a ring without zero divisor. Also find the units of Z_7, which is commutative ring with unity.

Ques 3. Every field is an integral domain. Prove this.

Ques 4. Prove that $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is a nilpotent element of $M_3(R)$, of real matrices of order 3x3.